REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. REPORT DATE *(DD-MM-YYYY)*	2. REPORT TYPE	3. DATES COVERED *(From - To)*
MAR 2012	Journal Article Post Print	OCT 2009 – SEP 2011

4. TITLE AND SUBTITLE

EXPERIMENTAL LIMITS ON LOCAL REALISM WITH SEPARABLE AND ENTANGLED PHOTONS

5a. CONTRACT NUMBER IN-HOUSE QIS0PROJ

5b. GRANT NUMBER N/A

5c. PROGRAM ELEMENT NUMBER N/A

6. AUTHOR(S)

Reinhard K. Erdmann, Michael L. Fanto, Paul M. Alsing, Corey J. Peters (AFRL/RITA)
Enrique J. Galvez (Colgate University, Hamilton, NY),
Warner A. Miller (Florida Atlantic University, Boca Raton, FL)

5d. PROJECT NUMBER QIS0

5e. TASK NUMBER PR

5f. WORK UNIT NUMBER OJ

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

AFRL/RITA
525 Brooks Road
Rome NY 13441-4505

8. PERFORMING ORGANIZATION REPORT NUMBER

N/A

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

AFRL/RITA
525 Brooks Road
Rome NY 13441-4505

10. SPONSOR/MONITOR'S ACRONYM(S)

N/A

11. SPONSORING/MONITORING AGENCY REPORT NUMBER

AFRL-RI-RS-TP-2012-005

12. DISTRIBUTION AVAILABILITY STATEMENT

APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED. PA #: 88ABW-2011-2390
DATE CLEARED: 28 APR 2011

13. SUPPLEMENTARY NOTES

14. ABSTRACT

Bell's theorem, and inequalities that stem from it, address the conflict between the explanation of key experimental observations by quantum mechanics (QM) and by models expressing Locally Realistic (LR) properties, regardless of their inclusion or exclusion of hidden variables. To demonstrate the conflict between experimental results described by QM and LR models, a physical realization of the quantum state must be chosen. Entangled photons or electrons provide the most viable choices. In this work we consider a simplified version of a Bell inequality (BI) that focuses entirely on the physical state properties of photons in order to demonstrate the difference between QM and LR correlations. While the experiment we propose is in principle similar in intent to prior Bell inequality experiments, our version requires fewer measurements, and is more advantageous in its conceptual clarity.

15. SUBJECT TERMS

polarization-entangled photons, Bell inequalities, local realism

16. SECURITY CLASSIFICATION OF: a. REPORT	b. ABSTRACT	c. THIS PAGE	17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
U	U	U	UU	14	PAUL M. ALSING

19b. TELEPHONE NUMBER (*Include area code*) N/A

Experimental limits on local realism with separable and entangled photons

Reinhard K. Erdmann, Michael L. Fanto, Paul M. Alsing, Corey J. Peters
Air Force Research Laboratory, Information Directorate, Rome, NY
Enrique J. Galvez
Colgate University, Hamilton, NY
Warner A. Miller
Florida Atlantic University, Boca Raton, FL

1. ABSTRACT

Bell's theorem, and inequalities that stem from it, address the conflict between the explanation of key experimental observations by quantum mechanics (QM) and by models expressing Locally Realistic (LR) properties, regardless of their inclusion or exclusion of hidden variables. To demonstrate the conflict between experimental results described by QM and LR models, a physical realization of the quantum state must be chosen. Entangled photons or electrons provide the most viable choices. In this work we consider a simplified version of a Bell inequality (BI) that focuses entirely on the physical state properties of photons in order to demonstrate the difference between QM and LR correlations. While the experiment we propose is in principle similar in intent to prior Bell inequality experiments, our version requires fewer measurements, and is more advantageous in its conceptual clarity.

Keywords: polarization-entangled photons, Bell inequalities, local realism

2. INTRODUCTION

The concept of real properties for matter and fields formed a cornerstone of physical theory long before more precise observations of atomic spectra and other phenomena fostered the development of QM to describe them. Although the intuition of realism is supported by virtually all macroscopic experiences, it cannot be sustained as a fundamentally correct description of nature. Taken as an approximation to an underlying QM description, however, it is entirely sufficient for most realms of experience. Specialized methods and efforts are required to observe quantum entanglement that unveils the lack of locally realistic (LR) properties. Such methods employing polarization-entangled photons have been used to demonstrate the conflict of LR properties with observations of nature. QM has proven considerably easier to describe than to explain; so the latter will not be attempted and the discussion in this work focuses on experimental settings where quantum entanglement manifestations and conflicts with local realism are optimal.

We interpret realism for objects or fields as properties possessed by objects that are independent of their measurement. These properties are assumed to provide a known response to any measurement, which may also be statistical in nature, thus entailing the use of probabilities. Such issues were dramatically highlighted in the 1935 Einstein-Podolsky-Rosen (EPR) paper [1] and by others. A series of insightful analyses were initiated by John Bell [2], largely in response to the issues raised by EPR and the apparent 'paradoxes' stemming from that work. The results of intensive BI experimental investigations have convincingly supported QM against all models based on LR or hidden variables [3, 4, 5, 6], even with a few experimental 'loopholes' remaining. For example, the 'efficiency loophole' requires a fair sampling assumption that is a particular challenge for photons, but progress continues toward its closure [7, 8]. To consider the QM-LR conflict in a simple and more direct context, we first describe an optimal manifestation of entanglement accessible to standard photon-based BI experimental configurations, such as illustrated in Figure 1. Entanglement means that certain correlation properties are associated with the composite bipartite photon state, but are not possessed by either member of the photon pair individually. For both QM and LR states, QM notation provides a convenient format to explicitly describe the inability for LR photon properties to describe the results of experiments employing states of entangled photons.

Quantum Information and Computation IX, edited by Eric Donkor, Andrew R. Pirich, Howard E. Brandt,
Proc. of SPIE Vol. 8057, 805707 · © 2011 SPIE · CCC code: 0277-786X/11/$18 · doi: 10.1117/12.883434

3. LR PHOTON STATES: THE CONFLICT

The simplest examples of photon states with LR properties are the separable states $|Y_1\rangle = |H\rangle|V\rangle$ and $|Y_2\rangle = |V\rangle|H\rangle$, where in $|Y_1\rangle$ the ket $|H\rangle$ denotes a polarized photon traveling to the left, and $|V\rangle$ denotes a vertically polarized photon traveling to the right. In both cases LR polarization properties are taken to be possessed by each photon. Their separation into product form ensures that there can be no influence by a measurement of say, the left photon upon the right photon, or vice versa, referred to as locality. We next consider an entangled state $|\Psi_{ent}^{(-)}\rangle = (|H\rangle|V\rangle - |V\rangle|H\rangle)/\sqrt{2}$ described in QM formalism as a coherent superposition of the two LR product states $|Y_1\rangle$ and $|Y_2\rangle$. The properties of $|\Psi_{ent}^{(-)}\rangle$ are however, entirely different from the LR product states $|Y_1\rangle$ or $|Y_2\rangle$. The entangled state $|\Psi_{ent}^{(-)}\rangle$ exhibits neither locality nor intrinsic ('real') polarization for individual photons. What the notation compactly expresses is that the two photons have opposite polarization regardless of which pair is manifested upon measurement. A detection of either the left or right member then determines the result for the other, e.g. if an H is detected on the left, it is certain that V will be on the right, and vice versa. The key distinction is that no polarization is assigned to either photon prior to detection. Because the detection of one photon inherently influences the result for the other, locality is violated. This is also expressed by the fact that the entangled state is not separable, i.e. it is not factorable into a product of two LR states, one component describing the left moving photon and the other describing the right moving photon.

Though the notation may set the stage, physical verification is required to demonstrate the existence of such non-local states lacking individual realism. This can be accomplished with the help of the experimental configuration in Figure 1. Figure 1 illustrates a typical BI experimental test setup, with the exception of optional polarizers inserted to generate (separable) polarization product states. The central source emits entangled-photon pairs via the physical process of spontaneous parametric down-conversion (SPDC). The physical properties of the emitted pairs that form indistinguishable degrees of freedom can in principle be entangled. However, in this work only the polarization degree of freedom of the photon is entangled; all others properties such as frequency or momentum are carefully unentangled in order to ensure there is no distinguishing polarization state information.

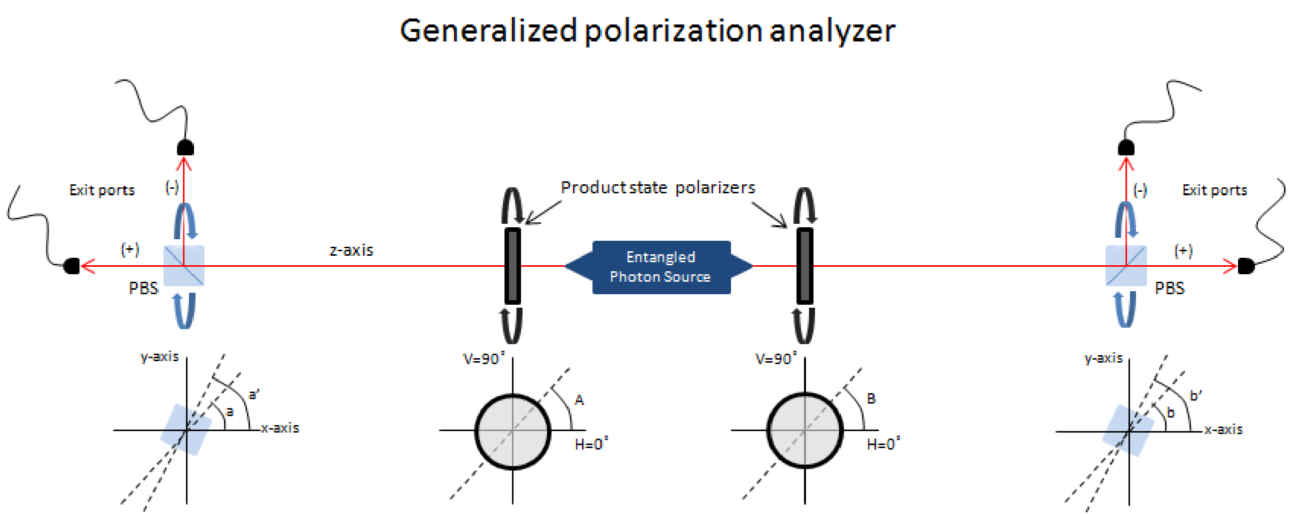

Figure 1. The experimental configuration depicted is similar to those used in BI tests that utilize entangled photons from an SPDC source. Product state polarizers are inserted to convert entangled photons to un-entangled states. Angles *a, a'* and *b, b'* define polarization beam splitter (PBS) detector angles at the left and right sides respectively, while *A, B* define polarization angles for the left/right moving product state photons. Horizontally polarized photons H correspond to 0° (aligned with *x* axis), and vertically polarized photons V correspond to 90° (aligned with y axis).

In the "classical" case (employing the product state polarizers in Figure 1) both right and left moving photon polarization states are well defined and correlated before measurement. Any global relationship between the right and left moving photons however, is completely precluded by the individual realism of the product state. This means that when measured in the same basis as the photon's polarization H,V, the result is perfect correlation i.e., a V (H) polarization at one detector implies that the other will be found with corresponding polarization H (V). There is clearly no mystery in this result until a different basis is chosen, and in particular when the choice of basis is made *after* the photons are emitted [6]. Both intuition and a straightforward calculation clearly reveal that the classical product state will not maintain perfect correlation for any basis not precisely aligned with the photon's polarization. As an example, at right/left detector settings of 45° the correlation becomes zero. Note that unlike experiments in [4] and [6], the set up in Figure 1 does not implement angle settings changes after the photons are emitted. Such a feature could be incorporated in the experimental setup of Figure 1, but closing the BI loopholes is not the focus of this present work and will not be further addressed.

In this work, we seek a clear distinction between measured results arising from entangled states and those constrained by LR properties. Such a distinction is not possible for single photon measurements. QM predicts that the polarization detected by means of measurements in the transmission (+) port or reflection (-) port of a polarization beam splitter (PBS) set to an arbitrary angle (basis) in Figure 1 will be random. The same statistical results can be simulated by a collection of LR states, linearly polarized at randomly selected orientations.

It could be expected that it is joint properties of the photon pairs that will offer the distinction sought for, manifested in the correlations that can be measured between the left and right photon pair members. Before defining such a correlation measure explicitly in the next section, we make the explicit assumption that all single-photon states considered obey Malus's Law at polarized interfaces, i.e. the transmission probability for a photon linearly polarized at angle A, at a linear polarizer detector with axis angle a, is given by $\cos^2(\text{A-a})$, with the corresponding probability for reflection (rejection) given by $\sin^2(\text{A-a})$, in terms of the local angle difference. We define a common (0°) reference for both detector PBS angles and photon polarization angles such that H = 0° and V= 90°. A general single-photon state of linear polarization can be given by $|\theta\rangle$ and a LR product state pair by $|\theta\rangle|\phi\rangle$. To progress, it will be necessary to consider measurements in bases set at angles other than 0° and 90° appropriate for H,V used thus far. This is accomplished by rotating the PBS before each photon counter, which allows each photon to be counted in one of the two ports; (+) for transmission, and (-) for reflection.

4. CORRELATION MEASUREMENTS: THE PROOF

The key measurements capable of distinguishing LR from QM, relate measurement results of a photon at one location (e.g. the left side of Figure 1) to those of the other photon at a spatially remote location (the right side of Figure 1). It will be evident that correlations derived from LR states will be quantitatively and characteristically different from those derived when entangled states are utilized. We define a correlation 'overlap' measure E with the following properties. If a photon enters the (+) port on the left, and the other photon also enters the (+) port on right, perfect correlation is defined by E=1. If the left/right photons enter opposite ports, say (+) for the left photon and (-) for the right photon, this measure is given by E = -1, describing perfect anti-correlation. Typically, the correlation data is statistically accumulated and the value of E is obtained by an average over many measurements at a given setting. An average value of E = 0 is designated as (perfectly) uncorrelated. Single-photon measurements are inherently statistical in that Malus's law yields only probabilities. Therefore, many experimental iterations are required to realize the algebraic form of the probabilities from the (+) and (-) product data accumulated at the four left/right port combinations: (+,+), (-,-), (+,-), and (-,+). In summary, a coincidence detection at the same ports on both sides (+,+) or (-,-) each yield a correlation value E = 1. A coincidence count (CC) at opposite ports (+,-) or (-,+) each yield a correlation value E= - 1. The net correlation is therefore the difference between the CCs in the same port and those in the opposite port, normalized by the total count number. Explicitly, E is given by the expression

$$E = \frac{N_{++} + N_{--} - N_{+-} - N_{-+}}{N_{++} + N_{--} + N_{+-} + N_{-+}} \tag{1}$$

where N_{ij} is the number of CCs for detecting photons in the *i*th port on the left and in the *j*th port on the right, with $i,j = \{+,-\}$. Note that the state is uncorrelated (E=0) if the number of CCs in the same ports is equal to the number of CCs in opposite ports.

Since the response for each single-photon state is known by Malus's law, E can be computed for any LR photon product state pair $|A\rangle|B\rangle$ with PBS detector angles set at *a, b* respectively. Each of the four coincidence combinations for the LR state yields a product of probabilities, with one factor corresponding to a probability for detection of the left photon, and the other factor corresponding to the probability for the detection of the right photon, as given by

$$\begin{aligned} &\mathrm{N}_{++} = \mathrm{Cos}^2(\mathrm{A\text{-}a})\mathrm{Cos}^2(\mathrm{B\text{-}b}), \qquad \mathrm{N}_{--} = \mathrm{Sin}^2(\mathrm{A\text{-}a})\mathrm{Sin}^2(\mathrm{B\text{-}b}), \\ &\mathrm{N}_{+-} = \mathrm{Cos}^2(\mathrm{A\text{-}a})\mathrm{Sin}^2(\mathrm{B\text{-}b}), \qquad \mathrm{N}_{-+} = \mathrm{Sin}^2(\mathrm{A\text{-}a})\mathrm{Cos}^2(\mathrm{B\text{-}b}). \end{aligned} \tag{2}$$

Note that each term in Eq. (2) is a product of sinusoidal terms whose arguments depend only upon local angle differences involving information solely on the left or solely on the right.

We can now examine the correlations arising from aforementioned states of particular interest, $|Y_1\rangle, |Y_2\rangle$ and $|\Psi_{ent}^{(-)}\rangle$. For $|Y_1\rangle = |H\rangle|V\rangle$, we let A= H = 0° and B = V = 90°. With some trigonometric manipulation we find for general PBS detector angles *a* and *b*:

$$E\left(Y_1(a,b)\right) = -\cos(2a)\cos(2b), \quad \text{Product State Pair Correlation,} \tag{3a}$$

$$E\left(\Psi_{ent}^{(-)}(a,b)\right) = -\cos\left(2(a-b)\right), \quad \text{Entangled State Correlation.} \tag{3b}$$

The second expression gives the well-known QM entangled state result (derived in the Appendix). The most pronounced difference between Eq. (3a) and Eq. (3b) is the fact that the entangled state correlation depends on the difference between the non-local detector angles (i.e. *a* on the left and *b* on the right), while the product state result depends on the product of terms whose arguments depend only upon local angle information. The origin of the angle difference *a-b* in the argument of Eq. (3b) stems the rotationally invariant (RI) nature of the entangled state $|\Psi_{ent}^{(-)}\rangle$. For the case of general polarizer/detector settings, one obtains $E\left(Y_1(a,b)\right) = -\cos(2[a-A])\cos(2[b-B+90^\circ])$ corresponding to Eq. (3a), while $E\left(\Psi_{ent}^{(-)}(a,b)\right) = -\cos\left(2[\{a-A\}-\{b-B+90^\circ\}]\right)$ generalizes Eq. (3b), clearly illustrating the non-local character of the entangled state correlation.

To realize other important correlation differences it is sufficient to consider the special case $a = b$ with both detectors at the same angle, which simplifies Eq. (3a) and Eq. (3b) to

$$E\left(Y_1(a,a)\right) = -\cos^2(2a), \quad \text{LR Correlation at } a = b, \tag{4a}$$

$$E\left(\Psi_{ent}^{(-)}(a,a)\right) = -1, \quad \text{Entangled Correlation at } a = b. \tag{4b}$$

The entangled state correlation described by Eq. (4b) is perfect (|E|=1) and independent of detector angle. This implies that the photons at each remote PBS detector always enter opposite polarization ports, even when the PBS detector angle settings (polarization bases) are chosen only after the photons are emitted from the source. A LR theory would have to address the question "how is this possible?" On the other hand, Eq. (4b) is a prediction of QM confirmed by experiments with entangled photons. A more interesting question to ask becomes: "Is Eq. (4b) realizable with photon states that are LR?" If the answer is "no," either conceptually or experimentally, the implication is that the assertion that photons possess LR properties must be invalid since it conflicts with the predictions of QM, as well as with experimental observation.

One counter example suffices in principle to establish the essential conflict. Consider the correlation of the real photon product states with identical detector angle settings (a = b) taking again A= H = 0° and B = V = 90°. For the case a=b = 0°, the correlations are perfect for both entangled and LR states

$$\left|E\left(\Psi_{ent}^{(-)}(0^\circ,0^\circ)\right)\right|=1, \quad \text{and} \quad \left|E\left(Y_1(0^\circ,0^\circ)\right)\right|=\left|E\left(Y_2(0^\circ,0^\circ)\right)\right|=1. \tag{5}$$

Eq. (5) shows that product states exhibit perfect correlation in a basis where detector angles are parallel (or orthogonal) to photon polarization. However, it is the correlation results from intermediate angle settings that provide the critical distinctions (as is also the case in violations of standard BI). A setting a=b = 45^0 reveals this most dramatically, yielding

$$\left|E\left(\Psi_{ent}^{(-)}(45^\circ,45^\circ)\right)\right|=1, \text{ and } \left|E\left(Y_1(45^\circ,45^\circ)\right)\right|=\left|E\left(Y_2(45^\circ,45^\circ)\right)\right|=0. \tag{6}$$

In Eq. (6) the entangled state retains perfect correlation at the new (in fact, at any a=b detector) setting, but the LR photon states become completely uncorrelated since in this case $E=-\cos^2(2\cdot 45^\circ)=0$. The analysis of the LR result can be readily explained; each LR photon enters a PBS detector on the left and right side with 50% probability to exit at each of the two ports independently. Thus, on average there will be an identical number of coincidences in the same ports as in opposite ports yielding E=0. The above examples motivate our development of an inequality to establish a limit on the correlations for LR photon pairs.

5. A LOCAL REALISM INEQUALITY FOR PHOTONS

We denote $E_{A,B}(a,b)$ as the correlation value for a product state $|A\rangle|B\rangle$ with left photon polarized at angle A, and right photon at angle B, for PBS detector angles a on left and b on right. We define a correlation parameter S_{LRP} (where LRP denotes 'locally realistic parameter' for photons)

$$S_{LRP} \equiv \left|E_{A,B}(0^\circ,0^\circ)+E_{A,B}(45^\circ,45^\circ)\right| \le 1, \tag{7}$$

analogous to a BI measure. The 'LRP inequality' expressed in Eq. (7) is a key result of this paper, and is operationally equivalent to previous BIs. In an actual experiment the left/right detector angles a,b would be (randomly) alternated between both angles set to 0° or both set to 45°. No LR product states (or statistical combination of them) can exceed the upper limit S_{LRP} =1 given in Eq. (7). However the correlations, found directly from employing Eq. (3b), confirm the entangled state $\left|\Psi_{ent}^{(-)}\right\rangle$ violates the LRP inequality by a factor of two (i.e. S_{LRP} =2), with perfect correlation (E=1) obtained for each term in Eq. (7) for detector settings a=b of 0° and 45°, respectively.

We have previously shown that the inequality in Eq. (7) is satisfied, at equality by the state $\left|Y_{0,90}\right\rangle \equiv |H\rangle|V\rangle=\left|0^\circ\right\rangle\left|90^\circ\right\rangle$. Although this state is only a single illustrative example, it will be seen that it is a limiting case. In fact, any two detector angles (a,b) would yield an inequality of the form of Eq. (7) with the different upper limit, but a 45° difference between the angles a and b yields the optimum contrast (i.e. the minimal upper limit in Eq. (7)). It can also be shown that the inequality in Eq. (7) holds for LR product states of polarization with complex amplitudes, as well as any incoherent combination of product states, so long as the weighting factor sums are properly normalized. Statistical results apply where individual detection events are not deterministic, such as each single photon's transmission at a polarizer. Eq. (7) also rules out the case where 'hidden variables' in the weighting factors could raise the averaged correlation value beyond the indicated upper limit.

Other differences and similarities of the measure S_{LRP} may be noted with respect to established BIs. The measure S_{LRP} differs by being confined to state properties observable in an experiment, whether conceptual or physical. The measurements would entail only two detector settings a and b, while in contrast a typical Clauser-Horne-Shimony-Holt

(CHSH) [3] Bell inequality test utilizes four angles a, a' and b, b'. Several BI tests have made use of only two settings [3,4]. The primary difference from the measures utilized in [3,4] and S_{LRP} is in the form; Eq. (7) involves a sum over two correlation (E) terms, while all prior BIs involve at least one difference term. The domains of distinction do not overlap, since the angle settings for which the LRP inequality is optimized (a=0°, b=0°, a'=45°, b'=45°) yield no CHSH violation for the entangled singlet state $\left|\Psi_{ent}^{(-)}\right\rangle$. Conversely, angle settings yielding maximal CHSH violation for the $\left|\Psi_{ent}^{(-)}\right\rangle$ do not pertain to the conditions of LRP inequality. There is therefore no conflict between results derived from our measure in Eq. (7) and that of the standard CHSH inequality, and the implications for local realism are essentially the same. The LRP inequality given in Eq. (7) is conceptually clear; LR product state photons can exhibit only half of the correlation (S_{LRP}=1) of an entangled state (S_{LRP}=2, in an optimal setting) as shown numerically in Figure 2, because the photons in the former state must respond independently at each side, while photons in the entangled state do not. Experiments to confirm the LRP inequality are technically analogous to Bell violations, but here the entangled state is measured only at perfect correlation settings. Such experimental results for the entangled state itself have been carried out in prior work, but not for the LR product states. In a CHSH violation the contrast is lower (2 for LR states and $2\sqrt{2}$ for maximally entangled states) in part because the more general scope of BI entails angle settings that cannot include perfectly correlated state measurements.

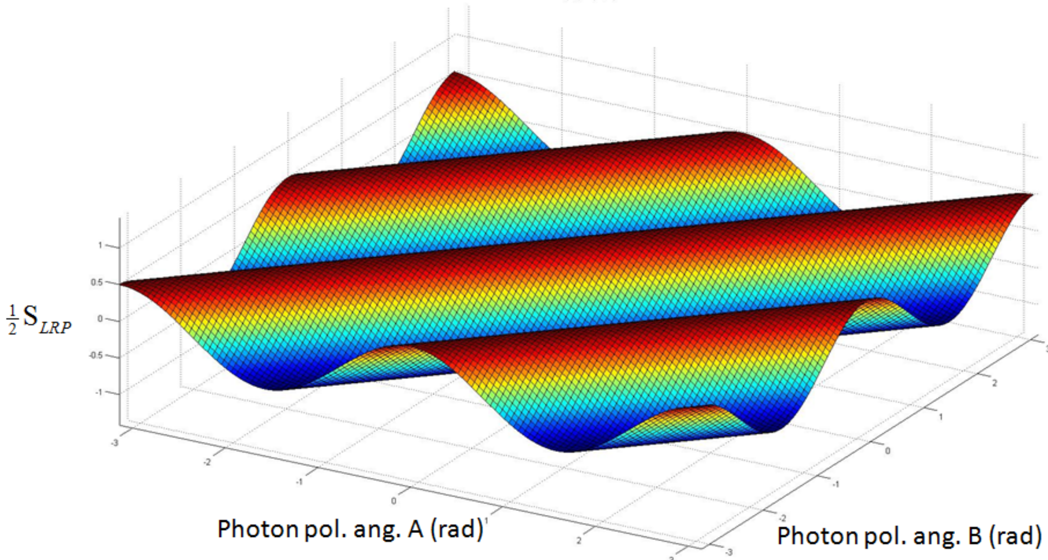

Figure 2. A plot of the local realism inequality parameter S_{LRP} of Eq. (7) shows that two-photon LR product states of linear polarization are bounded by S_{LRP} = 1.0. The entangled state $\left|\Psi_{ent}^{(-)}\right\rangle$ violates this inequality by a factor of 2 (S_{LRP} = 2.0).

5.1 Rotational Invariance for entangled and non-entangled photon pairs

From the discussion in Section 4, the RI nature of the entangled state $\left|\Psi_{ent}^{(-)}\right\rangle$ is evident in the quantum correlations in Eq. (3b) and Eq. (4b). Note however, that RI not assumed, nor necessary for the derivation of the LPR inequality in Eq. (7). By construction, RI is clearly not a property of the LPR states $\left|Y_1\right\rangle, \left|Y_2\right\rangle$ or in fact for most single-pair non-entangled states. The question to be posed is whether a generalized combination of such LR states could exhibit RI when the correlation measurements are taken as a statistical average, as occurs in standard Bell-type experiments. This means that the LR state under investigation may be any statistical combination of product states, such as $\left|Y_1\right\rangle$, and $\left|Y_2\right\rangle$. The (incoherent) sum is expressed as a density matrix of the form, say $\rho(Y_a) = \frac{1}{4}\rho(Y_1) + \frac{3}{4}\rho(Y_2)$, where the weight factors must sum to unity in order to normalize in order to conserver probability. By construction, this example LR state is not RI. The point here is to ascertain whether or not the most general LR photon pair state could mimic correlations produced by QM entangled states. Specifically, it is of direct interest to determine if a generalized LR state can be constructed to satisfy rotational invariance of the entangled state $\left|\Psi_{ent}^{(-)}\right\rangle$, and if so, can it reproduce the correlations o that quantum state?

The construction is straightforward, requiring only an equally-weighted contribution from all LR polarization state pairs. This resulting state involves a continuous sum that is conveniently expressed in integral form. Each component state is of the product state form $\left|\mathrm{Y}\right\rangle=\left|\theta\right\rangle\left|\phi\right\rangle$, or equivalently $\rho_Y=\left|\theta\right\rangle\left|\phi\right\rangle\left\langle\theta\right|\left\langle\phi\right|$ in a density matrix form. The state construction follows [9] using QM formalism that can treat either product or entangled states. The result is given below after adopting the simplification $\theta \perp \phi$ (or A=B+90° in Figure1) indicating that the source emits each photon pair with orthogonal polarization angles, which in an actual experiment would then be uniformly incremented. (The same statistics result if the angle settings are assumed to be randomized, though it is experimentally more difficult to implement). With the detector polarizers set to angles *a* and *b*, and denoting A(λ,a), B(λ,b) the probabilities of a (+) result at left and right respectively for each angle value of $\lambda = \theta$ (with $\phi = \lambda + 90^\circ$), the probability of coincidence detection in both (+) ports becomes

$$\mathrm{P}_{++} = \int \mathrm{p}(\lambda)\mathrm{A}(\lambda,\mathrm{a})\mathrm{B}(\lambda,\mathrm{b})\mathrm{d}\lambda \rightarrow \tfrac{1}{2\pi}\int_0^{2\pi} \cos^2(\lambda - a)\sin^2(\lambda - b)\,d\lambda = \tfrac{1}{4}\left(1 - \tfrac{1}{2}\cos 2(a-b)\right). \tag{8}$$

The first equality in Eq. (8) is equivalent to treating λ as a hidden variable (designating the LR polarization state), which is distributed uniformly over all angles with probability $p(\lambda) = 1/2\pi$. In the second integral, the *sin²* factor in the integrand arises from the orthogonal angle condition $\theta \perp \phi$, specifically chosen to mimic the RI of the entangled state as closely as possible. When the other joint probability terms are calculated similarly (see Appendix Eq. (A4)) the resulting correlation for a RI LR state is given by

$$E(Y_{RI}) = P_{++} + P_{--} - P_{+-} - P_{-+} = -\tfrac{1}{2}\cos\left(2(a-b)\right). \tag{9}$$

A comparison with the QM correlation $E\left(\Psi_{ent}^{(-)}(a,b)\right)$ in Eq. (3b) shows that the LR correlation in Eq. (9) is indeed identical in form, but the reduced in magnitude by a factor of two. This is fully consistent with the LR inequality (Eq. 7) and further clarifies the nature of the limits on LR, due to the independent nature of the response of the detectors upon measurement of each photon. In contrast, for the entangled state $\left|\Psi_{ent}^{(-)}\right\rangle$ the response of the detectors are not independent upon measurement of either photon, i.e. the left/right detection results are intrinsically correlated. The apparent non-local nature of the argument of Eq. (9) (involving the difference between spatially remote angles) is explained in the Appendix and shown to be intimately related to the factor of ½.

Another possible RI state is a product state constructed from circularly polarized single photon states $\left|\pm\right\rangle = \left|H\right\rangle \pm i\left|V\right\rangle$. However, such a state is not a particularly useful since its correlation is zero for any detector angle setting:

$$\left|Y_{CIRC}\right\rangle = \left|+\right\rangle\left|+\right\rangle \;\Rightarrow\; E(Y_{CIRC}) = 0. \tag{10}$$

This result arises from the cancellation of terms when Eq. (3a) is used to evaluate the four terms in the density matrix $\left|Y_{CIRC}\right\rangle\left\langle Y_{CIRC}\right|$ involved in the computation of $E(Y_{CIRC})$.

5.2 CHSH Inequality: LR Bounds for Photon States

The CHSH inequality defines a parameter S as

$$S = \left| E(a,b) + E(a',b) + E(a,b') - E(a',b') \right|, \tag{11}$$

in terms of the correlation measure E, used in the previous sections, and polarizer detector angles *a, a'* on the left and angles *b, b'* on the right (see Figure 1). It is well known that S ≤2 (with the upper limit S_{CL}=2 known as the 'classical bound') for all states expressing locally real properties including those possessing a hidden variable description; this is the assertion of the CHSH inequality. Additionally, it is also widely known that the entangled state $\left|\Psi_{ent}^{(-)}\right\rangle$ maximally violates the CHSH inequality, for certain detector angle choices (e.g. $\{a=\pi/4,\ a'=0,\ b=\pi/8,\ b'=-\pi/8\}$), at the value $S_Q = 2\sqrt{2}$, which is the upper (Tsirelson) quantum bound [10] achievable by any physically realizable state. The question may be posed "does the set of LR photon product states $|A\rangle|B\rangle$ saturate the classical bound S_{CL}=2 for the CHSH inequality?" A complete numerical search of linearly polarized pairs is straightforward with computational software such as Matlab, and the result is plotted in Figure 3. The simulation reveals that for the states considered the upper bound for S is in fact $\sqrt{2}$, rather than classical bound value of 2. It is indicative that the ratio between the maximal values of S in Figure 3 and S_Q is exactly ½ , the same value found in Eq. (9) established by the LR inequality in Eq. (7).

The CHSH inequality establishes upper limits on allowable states that are more general than those that pertain to LR photon-product states. The CHSH inequality establishes correct bounds for all states admitting a LR description, but S may not always achieve the value that saturates the inequality, S_{CL}=2. This may raise a question as to which upper bound should pertain in actual experiments when categorizing mixed states such as Werner states [11] which can exhibit CHSH values for the S parameter over the domain $\{0, 2\sqrt{2}\}$.

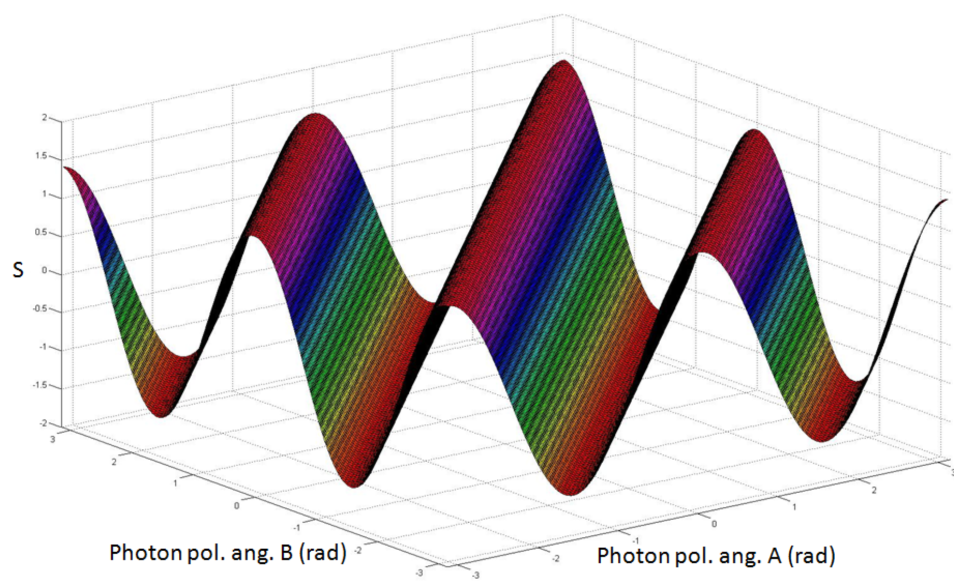

Figure 3. A plot of the S parameter in Eq. (11) for all LR product state pairs $|A\rangle|B\rangle$ of linear polarization. S is evaluated for the CHSH Inequality with detector angle settings that yield the maximum violation $2\sqrt{2}$ for the entangled photon state $\left|\Psi_{ent}^{(-)}\right\rangle$. The upper bound for S is $\sqrt{2}$ for these detector angle settings, versus its classically allowable maximum value of 2.

6. A CONCEPTUAL APPLICATION OF ENTANGLEMENT

The concept of quantum steering [12, 13] demonstrates the non-local effect that measurement has upon correlations. Quantum correlations, which violate local realism, enable a remote data acquisition that is not possible with LR photons.

A conceptual experiment entails several assumptions that are difficult to realize in a physically implementable experiment: (i) no photon losses are incurred anywhere except at the inserted polarizer due to Malus's law, and (ii) a single LR photon and an entangled-photon pair are available on demand. Nonetheless, Figure 4 illustrates the setup for a conceptual quantum steering experiment utilizing a single photon sent from Alice to Bob.

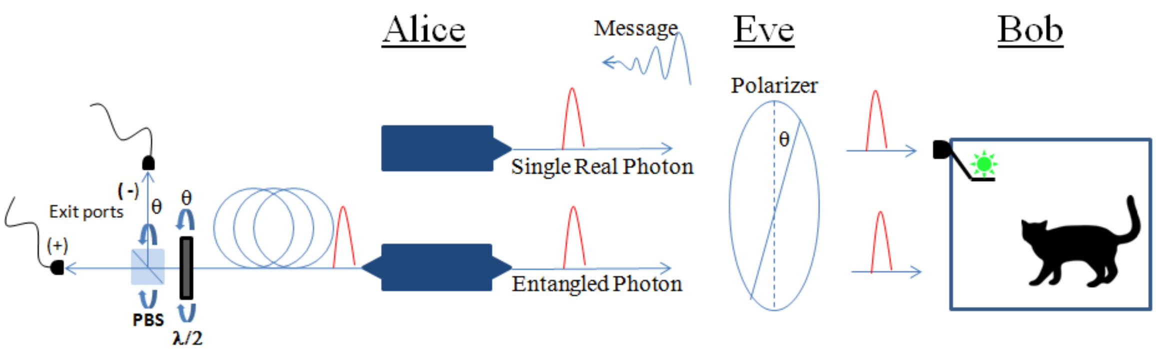

Figure 4. Quantum Steering of an Entangled-Photon State

Figure 4 illustrates that Alice, at the distant left, must send a signal to Bob (distant right) using only a single photon to determine the fate of Bob's cat, Schrodinger (a photon received by Bob triggers a device which acts questionably on the unsuspecting feline). There is only a single time slot available for the signal, which is known to Alice and Bob, as well as to an intermediate eavesdropper Eve.

6.1 Analysis for the case when Alice sends Bob a LR photon signal

In Figure 4, Eve (in the center) will disturb Alice's message photon to Bob by inserting a polarizer into the path at some angle orientation θ. Eve's objective is not to block the signal, but to let Alice know that its arrival will be uncertain. Eve therefore sends a classical message to Alice stating the polarizer orientation θ, carefully timed so the message arrives *after* Alice has launched her own signal photon towards Bob, but *before* it enters Eve's polarizer. Eve naturally assumes that Alice can do nothing except discern the probability for her photon's arrival at Bob.

6.2 Analysis for the case when Alice employs an entangled-photon signal

In this alternate case, Alice (knowing QM) acquires an SPDC source of polarization-entangled pairs $\left|\Psi_{ent}^{(-)}\right\rangle$. One photon is sent to Bob, and the other retained by Alice in a fiber loop (see Figure 4). When Alice receives the message from Eve revealing the perturbing polarizer angle θ, Alice (quickly) releases her fiber-loop-stored photon and passes it through a half-wave plate and into a PBS detector, both rotated to the angle θ. The measurement instantly collapses the entangled state before Alice's signal photon reaches Eve's polarizer. The signal photon sent to Bob therefore acquires the polarization state orthogonal to that of Alice's locally measured photon. Alice now knows with certainty (i.e. with unit probability) the state of Bob's received photon, in particular whether or not it passed Eve's polarizer. The details are: (i) if Alice measures her photon to be in the polarization state θ, then Bob receives no photon since the signal photon is in the state, $\theta_{\perp} = \theta + 90°$ and therefore does not pass through Eve's inserted polarizer set at angle θ, (ii) if Alice measures her photon to be in the state $\theta_{\perp}$ then Bob's signal photon must be in the orthogonal polarization state θ, and hence passes through Eve's inserted polarizer set at angle θ. Note that Alice has no control over which polarization state she measures, θ or $\theta_{\perp}$, because the collapse of the state in the port θ or $\theta_{\perp}$ is an inherently random process.

By utilizing a signal from an entangled-photon pair Alice therefore adapts to a specially contrived hostile intrusion by Eve, and is able to know with certainty the result of her photon message to Bob (and hence the fate of his cat). Alice is not able to control the outcome of the message received by Bob, as that would violate the no-signaling (non-superluminal) constraint of special relativity. Any source constrained by LR properties, such as the single LR photon utilized in the first case, could yield no definite information as to the fate of Bob's cat in this circumstance, rather only a probabilistic estimate. On the other hand, the non-local effect ('quantum steering') resulting from the use of the signal photon from an entangled-photon pair enables an elementary information processing task.

To complete this picture of measurement induced state collapse, it should be pointed out that Alice made a choice when she learned of the 'threat,' i.e. the value of θ; she could have instead waited until her signal photon entered Eve's polarizer before measuring her fiber-stored photon. The measurement-collapse would take place at Eve's polarizer, so a later measurement of Alice's stored photon (in the θ basis) reveals the result (fate) of Bob's photon (and hence, his cat). The final result in this later passive scenario appears to be similar to the end result of the active scenario discussed in Section 6.2 above. However, it turns out Alice had very good reason for her active choice in the 'steering.' By making use of entangled photons, Alice knows with unit probability the subsequent (remote) fate of Bob's cat; thus any consequential decisions on her part can be made with surety. Had Alice instead chosen the 'passive' option, the feline fate information would not be available to her until after her photon reaches the (very remote) intrusive polarizer, and she had made a measurement on her locally stored photon. With her 'active' choice, Alice induces the state collapse and can make use of the definite information regarding the feline's fate before it takes place (i.e. before her photon reaches Eve's polarizer). Clearly, with LR photons no such options are even conceivable.

7. SUMMARY

The implication that QM denies the existence of inherently real properties for individual objects was a concept unpalatable to A. Einstein [14]. The EPR paper [1] did not actually express disagreement with a QM description of nature, rather it suggested that a more complete theory could be found, and in particular one that would retain local realism. QM does not preclude LR properties for systems; conservation of energy, momentum and many other physical properties have unequivocally 'real' values in the theory prior to measurement. When entangled photons are utilized there is a 'joint reality' to the global description of the polarization ascribable to the composite singlet state of the pair of photons (analogous to zero spin), as opposed to the individual single photon constituents. What the work presented in this paper has explored, and which extensive prior research has confirmed, is that the notion of individual photon realism precludes certain correlations that are actually observed in experiments.

The QM description of an entangled system includes joint properties, but in general provides no description of individual 'real' properties ascribable to subsystems. It can therefore give correct descriptions of correlations which are joint properties, without conflicting with the constraints of local realism, a few of which have been explored in this work. If one were to seek a somewhat anthropomorphic interpretation, it could be argued that in certain systems where joint realism and individual realistic properties must be mutually exclusive, nature has chosen the former over the latter in the exhibition of entanglement. QM in general describes the results correctly without the need for such 'interpretation.' With regards to realism, Bell's work, along with that of many others, has clarified the notion of local realism. The work presented in this paper has attempted to develop a simple correlation measure to more directly assist in this clarification.

8. CONCLUSION

Bell's inequality and subsequent work convincingly established the case against LR and hidden variable models. That was accomplished by addressing all models that entail LR, and by using very general algebraic and logical arguments. Typically, violations of a BI are accomplished by means of a counterexample employing a quantum entangled state that is experimentally measured to demonstrate the maximal violation of the inequality. The approach taken in the work presented in this paper may be viewed as reversing the order: the entangled state properties are first examined in the context of the physical states measured experimentally. Those same states (photons in this case) are then constrained by LR properties to determine if there is a conflict with observed and predicted results. The scope is less general than prior BI analyses, and does not conflict with those results. Nevertheless, some new insight and experimental simplification is gained by the approach taken in this paper, and the quantitative results are in a sense complementary to the prior established work.

9. ACKNOWLEDGEMENTS

E.J.G. acknowledges support from RITC under grant FA8750-11-2-0034. W.A.M. acknowledges support from RITC under grant FA8750-10-2-0017. Any opinions, findings and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of AFRL.

10. REFERENCES

[1] Einstein A., Podolsky B., and Rosen N., “Can quantum mechanical description of physical reality be considered complete?”, Phys Rev **47** , 777 (1935).

[2] Bell J.S., “On the Einstein Podolsky Rosen Paradox”, Physics 1, 195 (1964).

[3] Clauser J.F., Horne M., Shimony A., and Holt R., “Proposed experiment to test local hidden-variable theories”, Phys. Rev. Lett., **23**, 880 (1969).

[4] Aspect A., Grangier P., and Roger G., “Experimental tests of realistic local theories via Bell’s Theorem”, Phys. Rev. Lett. **47**, 460 (1981).

[5] Genovese M., “Research on hidden variable theories; a review of recent progresses”, arXiv:quant-ph/0701071 v1, Jan (2007).

[6] Weihs G., Jennewein T., Simon C., Weinfurter H., and Zeilinger A., “Violation of Bell’s Inequality under strict Einstein locality conditions”, Phys. Rev. Lett. **81**, 5039 (1998).

[7] Garg A., Mermin N.D., “Detector Inefficiencies in the Einstein-Podolsky-Rosen experiment”, Phys. Rev. D **35**, 3831- 3835, (1987**).**

[8] Altepeter J.B., Jeffrey E.R., Kwiat P., Tanzilli S., Gisin N., and Acin A., “Experimental methods for detecting entanglement”, Phys. Rev. Lett. **95**, 033601 (2005).

[9] Kwiat P., “Effects of Entangled photons”, Ph.D. dissertation, Univ. Microfilm International, pg. 115, (1992) .

[10] Tsirelson, B.S., “Quantum Generalizations of Bell's Inequality,” Lett. Math. Phys. **4,** 93 (1980).

[11] Werner, R.F., “Quantum states with Einstein-Podolsky-Rosen correlations admitting a hidden-variable model,” Phys. Rev. A **40**, 4277 (1989).

[12] Schrodinger E., “Discussion of probability relations between separated systems”, Proc. Camb. Phil. Soc. **31,** 555 (1935).

[13] Saunders D.J., Jones S.J., Wiseman H.M., and Pryde G.J., “Experimental EPR-steering using Bell-local states”, Nature Physics **1766** Sept 19 (2010).

[14] In a letter to Max Born dated December 4, 1926, Einstein writes, “Quantum mechanics is certainly imposing. But an inner voice tells me that it is not yet the real thing. The theory says a lot, but does not really bring us any closer to the secret of the ‘old one.’ I, at any rate, am convinced that *He* does not throw dice**.”** *The Born-Einstein Letters* (translated by Irene Born), Walker and Company, New York, (1971).

APPENDIX: CORRELATION CALCULATIONS

A1. Correlation of the Entangled State $\left|\Psi_{ext}^{(-)}\right\rangle$

It should be noted that all calculations of transmission (reflection) at polarized interfaces (PBS), and correlations at several interfaces, can be calculated directly from Malus’s Law for either entangled or product states; QM formalism is convenient, but not essential to obtain the results. In the QM notation a linearly polarized photon state $|\theta\rangle$ can be defined by an angle in the $|H\rangle, |V\rangle$ basis. A polarization measurement entails projecting the state on to one of the two basis states with amplitude $\langle H \mid \theta\rangle$ and probability $|\langle H \mid \theta\rangle|^2 = \cos^2(\theta)$, expressing Malus’s Law. Rotation of the detector angle yields new basis elements $H_a, V_a = R_a\{H, V\}$ and $H_b, V_b = R_b\{H, V\}$ for left, right photons where

$$R_a = \begin{pmatrix} \cos(a) & \sin(a) \\ -\sin(a) & \cos(a) \end{pmatrix} \text{ and } R_b = \begin{pmatrix} \cos(b) & \sin(b) \\ -\sin(b) & \cos(b) \end{pmatrix}. \tag{A1}$$

The correlation probability for a photon pair at any polarized exit port combination is then given by: $P_{H,H} = \left|\langle H_a | \langle H_b | \Psi\rangle\right|^2$ with $P_{H,H}, P_{H,V}, P_{V,H}$ given analogously. The evaluation of E is straightforwardly carried out by insertion of any state into Eq. (10). However the evaluation of $E\left(\Psi_{a,b}\right) = P_{V,V} + P_{H,H} - P_{V,H} - P_{H,V}$ is simplified by utilizing the symmetry that arises from the rotational invariance of the entangled state. Using the rotation matrices it is straightforward to confirm that

$$R_a \oplus R_b(\Psi^{(-)}) = \tfrac{1}{\sqrt{2}}\left(\left|H_a\right\rangle\left|V_b\right\rangle - \left|V_a\right\rangle\left|H_b\right\rangle\right) \tag{A2}$$

has a form that is basis independent. This invariance under rotation allows a simple choice of basis for calculation; i.e. $a = 0$, and $\delta \equiv (a - b)$. Then, $H_a = H$ and $H_b = H_\delta$. Thus Eq. (10) and the three other terms can be evaluated by inspection to yield

$$\begin{aligned} E\left(\Psi^{(-)}_{a,b}\right) &= P_{V,V} + P_{H,H} - P_{V,H} - P_{H,V} \\ &= \tfrac{1}{2}\left|\left\langle V_\delta \mid V\right\rangle\right|^2 + \tfrac{1}{2}\left|\left\langle V_\delta \mid V\right\rangle\right|^2 - \tfrac{1}{2}\left|\left\langle V_\delta \mid H\right\rangle\right|^2 - \tfrac{1}{2}\left|\left\langle V_\delta \mid H\right\rangle\right|^2 \\ &= \tfrac{1}{2}\left(\sin^2(\delta) + \sin^2(\delta) - \cos^2(\delta) - \cos^2(\delta)\right), \\ \Rightarrow E\left(\Psi^{(-)}_{a,b}\right) &= -\cos\left(2(a-b)\right) \quad \text{for the entangled singlet state.} \end{aligned} \tag{A3}$$

A2. LR RI State

For the uniformly distributed (normalized) product state of Eq. (8) we have

$$\begin{aligned} P_{++}(a,b) &= \tfrac{1}{2\pi}\int_0^{2\pi} d\lambda \cos^2(\lambda - a)\sin^2(\lambda - b) = \tfrac{1}{4}[\tfrac{1}{2} - \cos 2(a-b)], \\ P_{--}(a,b) &= \tfrac{1}{2\pi}\int_0^{2\pi} d\lambda \sin^2(\lambda - a)\cos^2(\lambda - b) = \tfrac{1}{4}[\tfrac{1}{2} - \cos 2(a-b)], \\ P_{+-}(a,b) &= \tfrac{1}{2\pi}\int_0^{2\pi} d\lambda \cos^2(\lambda - a)\cos^2(\lambda - b) = \tfrac{1}{4}[\tfrac{1}{2} + \cos 2(a-b)], \\ P_{-+}(a,b) &= \tfrac{1}{2\pi}\int_0^{2\pi} d\lambda \sin^2(\lambda - a)\sin^2(\lambda - b) = \tfrac{1}{4}[\tfrac{1}{2} + \cos 2(a-b)], \end{aligned} \tag{A4}$$

which yields

$$E(Y_{RI}) = P_{+,+} + P_{-,-} - P_{+,-} - P_{-,+} = -\tfrac{1}{2}\cos\left(2(a-b)\right) \text{ for a LR state with Rotational Invariance.} \tag{A5}$$

A3. Origin of the apparent non-local argument and factor of ½ in Eq. (A5)

As noted in the text, Eq. (A5) has the same form as the correlation obtained from the entangled state $\left|\Psi^{(-)}_{ent}\right\rangle$ in Eq. (3b), but with ½ the magnitude. In addition, the argument of cosine in Eq. (A5) apparently exhibits non-local behavior in that it involves the difference of remotely separated detector angles: *a* on the left and *b* on the right. This later observation is in fact intimately related to the factor of ½ in Eq. (A5) as can be seen as follows.

From the expression of $E(Y_{RI})$ in Eq. (A5), let us first sum the integrands in Eq. (A4) before performing the integral over the angle λ. Grouping terms, we find that the integrand of the expression $(P_{+,+} - P_{+,-}) + (P_{--} - P_{-+})$ is given by – cos(2(a- λ)) cos(2(b- λ)) which exhibits local realism in that the expression factors into a product of two terms, whose arguments depend only on local angle differences solely on the left and solely on the right, respectively. Using a trigonometric identity, we can rewrite this expression as the sum of two cosine terms whose arguments are the sum and difference of the local angle differences (*a*- λ) and (*b*- λ) yielding - ½ cos(2({a- λ }-{ b- λ })) – ½ cos(2({a- λ }+{ b- λ })) = -½ cos(2(a-b)) -½ cos(2(a+b-2 λ)). The first term -½ cos(2(a-b)) is independent of photon angle λ and is unaffected by the subsequent integration over λ. This is precisely the expression appearing in Eq. (A5), which is half the magnitude of the quantum entangled state correlation $E\left(\Psi^{(-)}_{ent}(a,b)\right)$ given by Eq. (3b). The remaining term -½ cos(2(a+b-2 λ)) explicitly depends on λ and therefore averages out upon the subsequent integration over λ.

The conclusion is that both the factor of ½ and the remote angle difference form (*a-b*) of the argument of Eq. (A5) both stem from two sources: (i) a trigonometric re-writing of the LR factorable form – cos(2(a- λ)) cos(2(b- λ)) for the argument of the correlation for the state Y_{RI} , and (ii) our particular experimental choice to set the left and right moving photon angles to the same value $A=B=\lambda$, which forces one half of expression in (i) to average out upon the integration over λ (required for expressing the RI fo the LR state Y_{RI}).